Appendix A

This appendix includes the solutions and answers to end of segment self-assessment problems and questions.

Segment 1 – Solutions

1. A plating tank with an effective resistance of 100 ohm is connected to the output of a full-wave rectifier. The AC supply voltage is $340V_{peak}$. Determine the amount of time, in hours, it would take to perform 0.075 faradays worth of electroplating?

Solution:
Therefore, the amount of charge transfer, in Coulombs, in this electro deposition case, would be:

$$q = \left(\frac{96{,}487 \text{ Coulombs}}{1 \text{ Faraday}}\right) . (0.075 \text{ Faraday}) = 7237 \text{ Coulombs}$$

As explained in Example 1.1, we are interested in the amount of time it takes to transfer a known amount of charge, so, rearrangement of Eq. 1.1 results in:

$$t = \frac{q}{I}$$

The next step entails determination of the DC voltage and current produced by the full wave rectification of 340 V_{peak} AC; which is the same as $340V_{max}$.

$$V_{max} = \sqrt{2}.\, V_{rms} = 340 \text{ V}$$

$$V_{DC} = 2.\left(\frac{V_{max}}{\pi}\right) = 2.\left(\frac{340}{3.14}\right) = 216 \text{ V} \qquad \textbf{Eq. 1.2}$$

And,

$$I_{DC} = \frac{V_{DC}}{R} = \frac{V_{Ave}}{R} = \frac{216}{100} = 2.16\text{A} \qquad \textbf{Eq. 1.4}$$

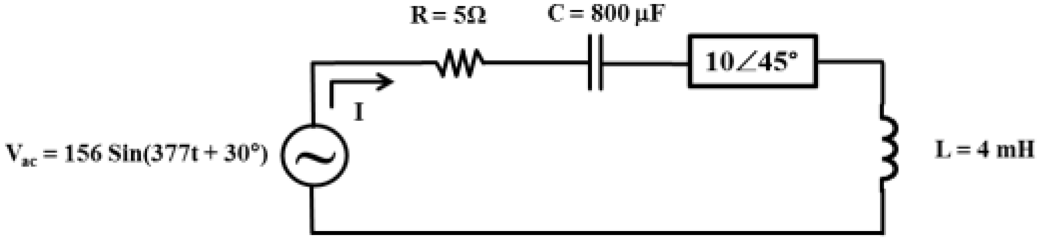
Then, application of Eq. 1.1 yields:

$$t = \frac{7237 \text{ Coulombs}}{2.16 \text{ Coulombs/sec}} = 3350 \text{ seconds}$$

Or,

$$t = \frac{3350 \text{ seconds}}{3600 \text{ second/hr}} = 0.93 \text{ hours}$$

2. Determine the source current I_{rms} in the AC circuit below.



Solution:

Solution strategy: Convert the given inductance value of L = 4 mH into its equivalent inductive reactance **X_L**. Convert the given capacitance value C = 800μF into its equivalent capacitive reactance **X_C**. Convert the given "black box" impedance of **Z=10∠45°** Ω into the rectangular form. Combine **R, X_L, X_C** and **Z,** linearly, to determine **Z_{EQ}**. Then apply Ohm's Law to calculate the source current **I**. Note that the given polar version of the "black box" impedance **Z=10∠45°Ω** can be converted into the equivalent rectangular form through Pythagorean Theorem, as explained in this segment, or through a scientific calculator to:

$Z = 10\angle 45° = 7.07 + j7.07\Omega$

$\therefore\ Z_{Eq} = R + Z_C + Z_L + Z$

$= R - jX_C + jX_C + Z$

$= R - j\dfrac{1}{\omega C} + j\omega.L + Z$

$= R - j\dfrac{1}{(2).(\pi).(f).(C)} + j(2).(\pi).(f).(L) + Z$

$= 5 - j\dfrac{1}{(2).(3.14).(60).(800x10^{-6})} + j.(2).(3.14).(60).(4x10^{-3}) + 7.07 + j7.07$

$= 5 - j\dfrac{1}{0.301} + j1.51 + 7.07 + j7.07$

$= 5 - j3.32 + j1.51 + 7.07 + j7.07$

$= 12.07 + j5.26 = 13.2\angle 23.55°\Omega$

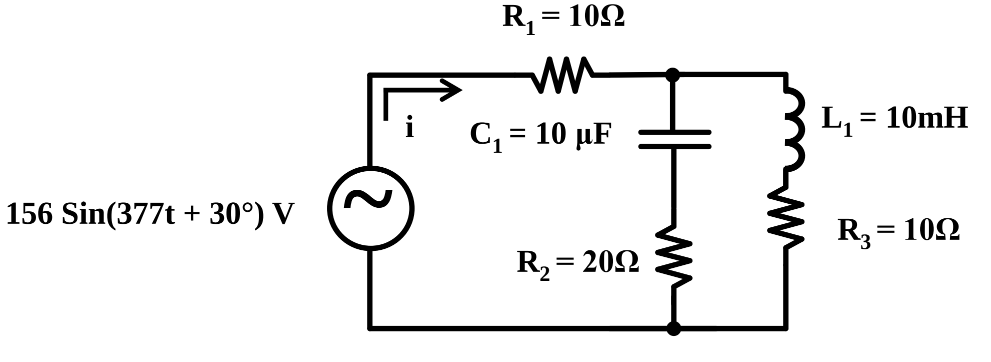
Current **"I"** calculation:

V(t), in rms form $= 110\ \angle 30° V_{rms}$ as determined in the chapter examples.

Then, according to Ohm's Law, $I = \dfrac{V}{Z_{Eq}} = \dfrac{110\ \angle 30°}{13.17\angle 23.55}$

$\therefore$ The source rms current, $I = 8.35\ \angle 6.5° A$

3 Calculate the impedance $\mathbf{Z_{EQ}}$ as seen by the AC voltage source in the circuit below:



Solution:

Solution strategy: Convert the given inductance value of L_1 = 10 mH into its equivalent inductive reactance X_L. Convert the given capacitance value C_1 = 10μF into its equivalent capacitive reactance X_C. Combine the load elements in each of the two parallel branch circuits into a single branch circuit impedance, and then combine the resulting parallel branch circuits into a single impedance, using parallel load combination formula. The impedance thus derived would be combined, through series combination approach, with resistor R_1=10Ω to arrive at the combined equivalent impedance representing all load elements driven by the 156 V_P AC source.

The 10mH inductor and the 10Ω resistive branch circuit, series combination:

$$
\begin{aligned}
Z_{R_3L} &= R_3 + Z_L \\
&= R_3 + jX_L \\
&= R_3 + j\omega.L \\
&= R_3 + j(2).(\pi).(f).(L) \\
&= 10 + j.(2).(3.14).(60).(10x10^{-3}) \\
&= 10 + j3.77 = 10.7\angle 20.7°\Omega
\end{aligned}
$$

The 10μF capacitor and the 20Ω resistive branch circuit, series combination:

$$
\begin{aligned}
Z_{R_2C} &= R_2 + Z_C \\
&= R_2 - jX_C \\
&= R_2 - j\frac{1}{\omega C} \\
&= 20 - j\frac{1}{(2).(3.14).(60).(10x10^{-6})} \\
&= 20 - j265.4 \quad = 266\angle -85.7°\Omega
\end{aligned}
$$

The formula for parallel combination of the RC and RL parallel circuits:

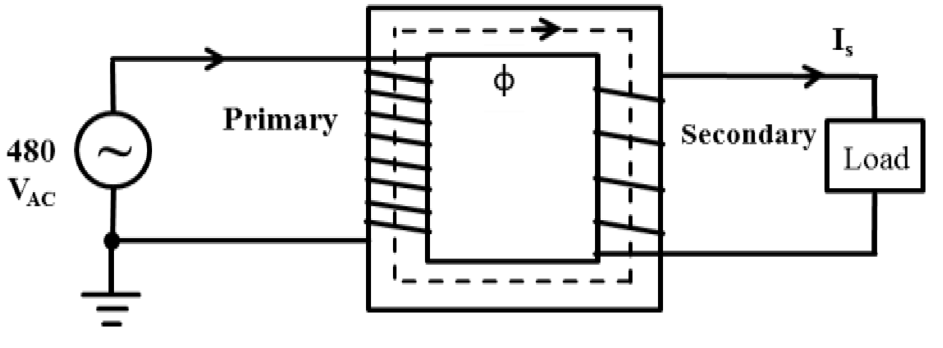
$$\frac{1}{Z_{EqRLC}} = \frac{1}{Z_{R_2C}} + \frac{1}{Z_{R_3L}}$$

$$\text{Or, } Z_{EqRLC} = \frac{(266\angle\text{-}85.7).(10.7\angle 20.7)}{266\angle\text{-}85.7 + 10.7\angle 20.7} = \frac{2848\angle\text{-}65°}{263.3\angle\text{-}83.5°} = 10.8\angle 18.5°$$

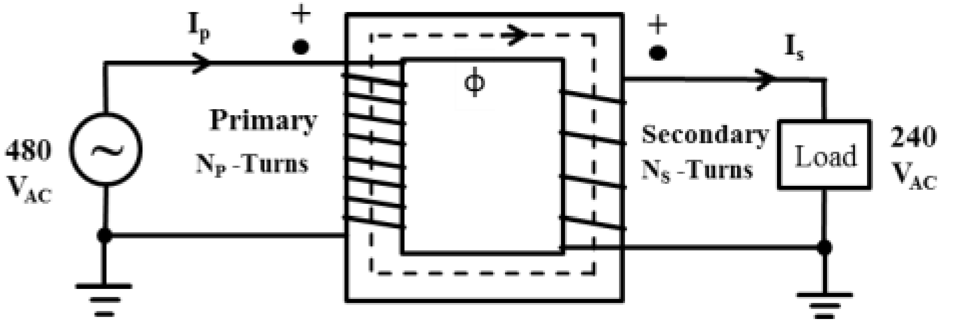
The final combination consisting of $R_1 = 10\Omega$ and the newly derived impedance $Z_{EqRLC} = 10.8\angle 18.5\Omega$:

$$\begin{aligned} Z_{Eq} &= R_1 + Z_{EqRLC} \\ &= 10 + 10.8\angle 18.5 = 10 + 10.26 + j3.43 = 20.26 + j3.43 \\ &= 20.55\angle 9.6\Omega \end{aligned}$$

4. A single phase 1kVA resistive load, designed to operate at 240 VAC, has to be powered by a 480 VAC source. A transformer is applied as shown in the diagram below. Answer the following questions associated with this scenario:
(a) Would the transformer be connected in a “step up” configuration or a “step down” configuration?
(b) When installing the transformer, what turns ratio should it be connected for?
(c) What would be the secondary current, I_s, when the load is operating at full capacity?
(d) What would be the primary current at full load?



Solution:
The scenario described in this problem can be illustrated as follows:



(a) As discussed in this segment, when voltage must be reduced to accommodate the load, the transformer must be connected in a step down configuration. Therefore, the answer is: **step down configuration**.

(b) **Turns ratio:** According to Eq. 1.3, the relationship between the turns ratio, the primary voltage and the secondary voltage is defined, mathematically, as:

$$V_s = \frac{V_p}{a} = \left(\frac{N_s}{N_p}\right).V_p \qquad \text{Eq. 1.12}$$

Or,

$$a = \frac{V_p}{V_s}$$

Therefore,

$$\text{Turns Ratio} = a = \frac{480}{240} = 2{:}1$$

(c) **Secondary current:** As explained the single phase transformer section of this segment, and mathematically expressed in form of Eq. 1.15:

$$\text{Single Phase AC Apparent Power} = \left|S_{1-\Phi}\right| = V_s.I_s = V_p.I_p$$

Rearranging of this equation yields:

$$I_s = \frac{\left|S_{1-\Phi}\right|}{V_s}$$

Or,

$$\text{Secondary Current} = I_s = \frac{|S_{1-\Phi}|}{V_s} = \frac{1000\ \text{VA}}{240\text{V}} = 4.2\ \text{A}$$

(d) **Primary current at full load:** According to ideal transformer equation Eq. 1.4,

$$I_s = a.I_p = \left(\frac{N_p}{N_s}\right).I_p$$

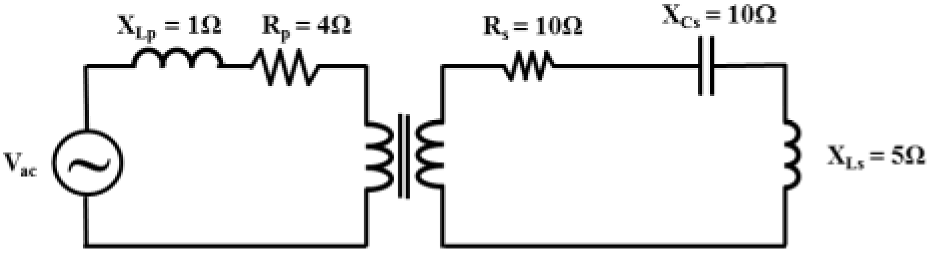
Or,

$$I_p = \frac{I_s}{a}$$

Since the turns ratio was determined to be 2:1 in part (b), and the secondary current I_s was determined to be 4.2 A in part (c),

$$I_p = \frac{I_s}{a} = \frac{4.2}{2} = 2.1\ \text{A}$$

5. Calculate the equivalent impedance as seen from the vantage point of the AC source V_{ac} in the circuit shown below. The transformer in the circuit is assumed to be ideal. The values of the primary and secondary circuit elements are: $X_{lp} = 1\Omega$, $R_p = 4\Omega$, $R_s = 10\Omega$, $X_{Ls} = 5\Omega$, $X_{Cs} = 10\Omega$, $N_p = 100$, $N_s = 200$.



Solution:

As in Example 1.2, for simplicity, the primary and secondary inductors are given in form of their reactances "X_L" instead of inductance values. In this problem, a capacitor is introduced on the secondary side. This capacitor is represented in form its X_{Cs}. As explained earlier in this segment, in the process of reducing the AC circuits to an equivalent impedance form, the individual inductive and capacitive reactances are combined in form of their respective impedance.

Therefore, on the primary side:

$\mathbf{Z_{Lp}} = jX_{Lp} = j1\Omega$

Hence, $\mathbf{Z_P} = R_p + \mathbf{Z_{Lp}} = 4 + j1\Omega$

On the secondary side:

$\mathbf{Z_{Ls}} = jX_{Ls} = j5\Omega$, and

$\mathbf{Z_{Cs}} = -jX_{Cs} = -j10\Omega$; the reason for the negative sign, as explained in Segment 1, lies in the fact that $\mathbf{Z_{Cs}} = (1/\mathbf{j})X_{Cs}$ and that $1/\mathbf{j} = \mathbf{-j}$

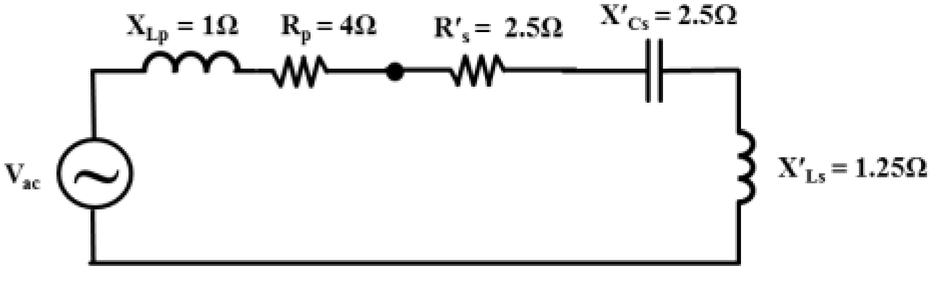
Hence, the total secondary impedance = $\mathbf{Z_s} = R_s + \mathbf{Z_{Ls}} + \mathbf{Z_{Cs}}$

$= 10 + j5 - j10\Omega$

***a* =** Turns ratio = $\mathbf{N_p/N_s} = 100/200 = 1/2$

$\boldsymbol{Z'_s} = \mathbf{a^2}.\boldsymbol{Z_s}$

$\boldsymbol{Z'_s} = \mathbf{(1/2)^2}.\ (10 + j5 - j10\Omega) = 2.5 + j1.25 - j2.5\Omega$

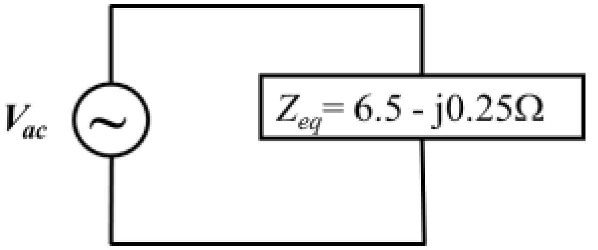


Then, combination of the primary impedance and the total reflected secondary impedance would result in the equivalent impedance $\mathbf{Z_{eq}}$:

$\boldsymbol{Z_{eq}} = \mathbf{Z_p} + \boldsymbol{Z'_s} = (4 + j1\Omega) + (2.5 + j1.25 - j2.5\Omega)$

$= \mathbf{6.5 - j0.25\Omega}$

This final equivalent impedance $\mathbf{Z_{eq}}$ can be represented in rectangular complex form as:



The equivalent impedance **Z_{eq},** derived in the rectangular form above, can be stated in polar or phasor form as:

$\boldsymbol{Z_{eq}}$**=** 6.5 – j0.25Ω = **6.5∠-2.2°**

This conversion from rectangular to phasor form can be accomplished through a scientific calculator, with complex math feature or, as illustrated earlier in this segment, through application of Pythagorean Theorem and trigonometry.

6. The no load voltage at the main switch yard of a manufacturing facility is 13,400 V_{AC}. The voltage regulation of the main switch yard is 4%. What is the rated full load voltage that is most likely to be measured on the load side of the main switch yard.

Solution:

$\mathbf{V_{NL}}$ = 13,400 V
Voltage Regulation **= 4% = 0.04**
$\mathbf{V_{FL}}$ = $\mathbf{V_{Rated}}$ = ?

Apply Eq. 1.19:

$$\text{Voltage Regulation in \%} = \left[\frac{V_{NL} - V_{FL}}{V_{FL}}\right] \text{x } 100$$

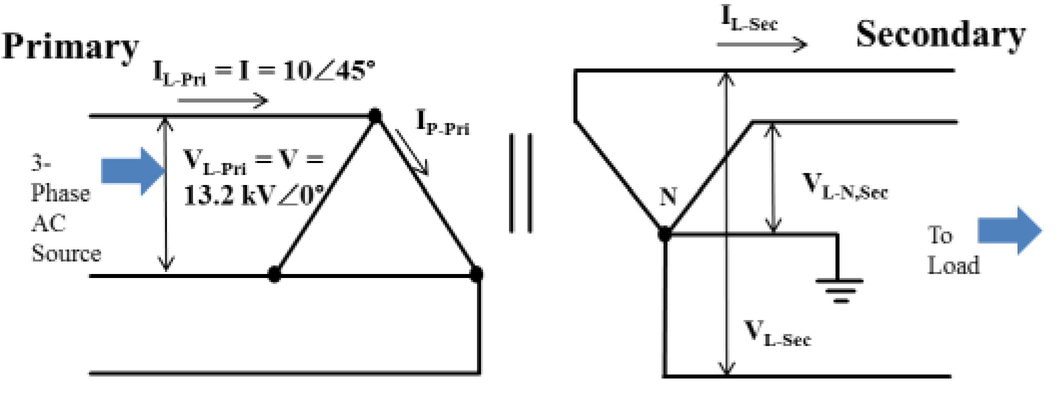
$$\text{Voltage Regulation in \%} = 4 = \left[\frac{13{,}400 - V_{FL}}{V_{FL}}\right] \text{x } 100$$

$$0.04V_{FL} = 13{,}400 - V_{FL}$$

$$V_{FL} = \frac{13{,}400}{1.04} = 12{,}885\ V_{AC}$$

7. Consider the power distribution system shown in the schematic below. Determine the following unknown parameters on the Y load side of the transformer given that the turns ratio is 2:1:

- **a)** $|\mathbf{I}_{\mathbf{L\text{-}Sec}}|$ = Magnitude of load or secondary line current
- **b)** $|\mathbf{I}_{\mathbf{P\text{-}Sec}}|$ = Magnitude of secondary phase current or load phase current
- **c)** $|\mathbf{V}_{\mathbf{P\text{-}Pri}}|$ = Magnitude of Phase voltage on the source or primary side of the transformer
- **d)** $|\mathbf{V}_{\mathbf{L\text{-}Sec}}|$ = Magnitude of Line voltage on the load or secondary side of the transformer
- **e)** $|\mathbf{V}_{\mathbf{P\text{-}Sec}}|$ = Magnitude of Phase voltage on the load or secondary side of the transformer
- **f)** $|\mathbf{V}_{\mathbf{L\text{-}N,\,Sec}}|$ = Magnitude of Line to neutral voltage on the load or secondary side of the transformer



Solution:

$$V_{P\text{-}Pri} = V_{L\text{-}Pri} \qquad \text{Eq. 1.28}$$

$$V_{L\text{-}Sec} = V_{P\text{-}Sec} \cdot \sqrt{3} \qquad \text{Eq. 1.29}$$

$$I_{P\text{-}Pri} = \frac{I_{L\text{-}Pri}}{\sqrt{3}} = \frac{I}{\sqrt{3}} \qquad \text{Eq. 1.30}$$

$$I_{P\text{-}Sec} = I_{L\text{-}Sec} \qquad \text{Eq. 1.31}$$

Where,

$\mathbf{V_{P\text{-}Pri}}$ = Primary phase voltage = **V**
$\mathbf{V_{L\text{-}Pri}}$ = Primary line voltage = $V_{Line\text{-}Delta}$ = **V**
$\mathbf{V_{P\text{-}Sec}}$ = Secondary phase voltage
$\mathbf{V_{L\text{-}Sec}}$ = Secondary line voltage = $V_{Line\text{-}Y}$

$\mathbf{I_{L\text{-}Pri}}$ = Primary line current = **I**
$\mathbf{I_{P\text{-}Pri}}$ = Primary phase current
$\mathbf{I_{P\text{-}Sec}}$ = Secondary phase current = $\mathbf{I_{Line\text{-}Y}}$
$\mathbf{I_{L\text{-}Sec}}$ = Secondary line current = $\mathbf{I_{Line\text{-}Y}}$

For a Δ-Y three phase transformers, as illustrated in figure 3.26, the voltage and current transformations can be assessed using the following equations:

$$\frac{V_{Line\,\text{-}\,Y}}{V_{Line\,\text{-}\,Delta}} = \frac{\sqrt{3}}{a}$$

$$\text{or,}\ \ V_{Line\,\text{-}\,Y} = \frac{\sqrt{3}}{a}.V_{Line\,\text{-}\,Delta} = \frac{\sqrt{3}}{a}.V \qquad \text{Eq. 1.32}$$

$$\frac{I_{Line\,\text{-}\,Y}}{I_{Line\,\text{-}\,Delta}} = \frac{a}{\sqrt{3}}$$

$$\text{or,}\ \ I_{Line\,\text{-}\,Y} = \frac{a}{\sqrt{3}}\ .I_{Line\,\text{-}\,Delta} = \frac{a}{\sqrt{3}}\ .I \qquad \text{Eq. 1.33}$$

a) According to Eq. 1.33:

$$I_{Line\,\text{-}\,Y} = \frac{a}{\sqrt{3}}\ .I_{Line\,\text{-}\,Delta} = \frac{a}{\sqrt{3}}\ .I$$

$$\text{or,}\ \left|I_{L\,\text{-}\,Sec}\right| = \frac{a}{\sqrt{3}}\ .\left|I\right| = \frac{2}{\sqrt{3}}\ .\left|10\angle 45^\circ\right| = \frac{20}{\sqrt{3}} = 11.55\ \text{A}$$

b) As characteristic of Y three phase AC circuits and in accordance with Eq. 1.31

$I_{P\text{-}Sec} = I_{L\text{-}Sec}$

or, $|I_{P\text{-}Sec}| = |I_{L\text{-}Sec}| = 11.55\text{ A}$

c) As characteristic of Δ three phase AC circuits and in accordance with Eq. 1.28

$V_{P\text{-}Pri} = V_{L\text{-}Pri}$

and, $|V_{P\text{-}Pri}| = |V_{L\text{-}Pri}|$

$\therefore\ |V_{P\text{-}Pri}| = 13.2\text{ kV}$

d) According to Eq. 1.32:

$$V_{Line\text{ - }Y} = \frac{\sqrt{3}}{a}.V_{Line\text{ - }Delta} = \frac{\sqrt{3}}{a}.V$$

$$\text{or, } |V_{L\text{ - }sec}| = \frac{\sqrt{3}}{a}.|V| = \frac{\sqrt{3}}{2}.|13.2\angle 0°| = 11.432\text{ kV} = 11{,}432\text{ V}$$

e) As shown in figure 3.26, on the secondary side of the transformer:

$$V_{P\text{ - }Sec} = \frac{V_{Line\text{ - }Delta}}{a} = \frac{V}{a}$$

$$\text{and, } |V_{P\text{ - }sec}| = \frac{|V|}{a} = \frac{|13.2\angle 0°|}{2} = 6.6\text{ kV} = 6{,}600\text{ V}$$

f) As apparent from figure 3.25 (a):

$V_{L\text{- }N,\text{ }Sec} = V_{P\text{-}Sec}$

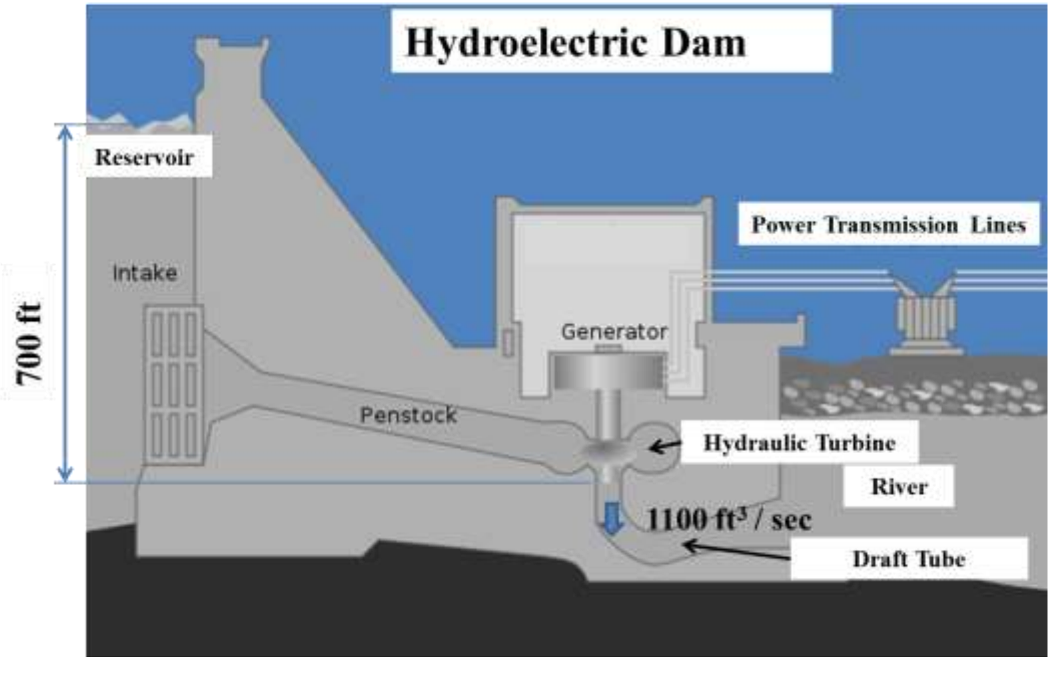
$\therefore\ |V_{L\text{- }N,\text{ }Sec}| = |V_{P\text{-}Sec}| = 6.6\text{ kV or }6{,}600\text{ V}$

Segment 2 - Solutions

1. Consider a hydroelectric reservoir where water is flowing through the turbine at the rate of 1100 ft^3 / sec. See the diagram below. The turbine exit point is 700 ft lower than the elevation of the reservoir water surface. The turbine efficiency is 90% and the total frictional head loss through the penstock shaft and turbine system is 52 ft.

a) Calculate the power output of the turbine in MW's.

b) If the efficiency of the Electric Power Generator is 92%, what would the electric power output be for this hydroelectric power generating system?



Solution

Given:

γ = specific weight = 62.4 lbf / ft^3

$\mathbf{h_f}$ = Frictional head loss in the penstock and elsewhere in the system, upstream of the turbine = 52 ft

$\mathbf{H_Z}$ = Total head available due to the height of the water level in the reservoir = 700 ft

$\therefore$ Net head delivered by the water to the turbine = Head added

$\mathbf{= h_A = H_Z - h_f}$ = 648 ft

$\dot{V}$ = Volumetric Flow Rate = 1100 ft^3 / s

Turbine Efficiency 90 %
Generator Efficiency 92 %

a) Calculate the power output of the turbine in MW's.

$$P_p = \text{WHP} = \text{Hydraulic Horsepower in hp} = \frac{h_A . \gamma . \dot{V}}{550} \qquad \textbf{Eq. 2.39}$$

$$= \frac{(648)(62.4)(1100)}{550} = 80{,}870\text{hp}$$

P_{out} = Power Output of the Turbine
= (0.90 x 80,870 hp)
= 72,783 hp

$$P_{out} = \text{Power Output of the Turbine in MW} = \frac{72{,}783\text{hp x } 0.746 \text{ kW/hp}}{1000 \text{ kW/MW}}$$

$$= 54 \text{ MW}$$

b) If the efficiency of the Electric Power Generator is 92%, what would the electric power output be for this hydroelectric power generating system?

$P_{\text{Gen. Syst.-out}}$ = Power Output of the Generator in MW
= (Power Output of the Turbine in MW).(η_{Turbine})
= (54 MW x 0.92)
= 50 MW

2. Which of the following two water heaters would cost the least to operate, on annual cost basis, under the given assumptions?

A. Electric Water Heater:
Estimated annual energy required to heat the water: 9,000 kWh
Efficiency**: 95%**
Cost Rate: **$0.10/kWh**

B. Natural Gas Water Heater:
Estimated annual energy required to heat the water: Same as the Electric water heater
Efficiency: **98%**
Cost Rate**: $10.87/DT**

Solution:

Note the given 9000 kWh worth of electrical energy is the ***energy actually absorbed by the water.*** Since the electric water heater efficiency is 95%, the electrical energy ***pulled from the utility*** would be:

$$= \left(\frac{9000 \text{ kWh}}{0.95} \right) = 9{,}474 \text{ kWh}$$

And the annual operating cost for the electric water heater would be:

$$= (9{,}474 \text{ kWh}).\left(\$0.10/\text{kWh} \right) = \$947.40$$

The ***annual energy absorbed by the gas water heater***, assumed to be the same as the electric water heater = 9000 kWh. Since the gas water heater efficiency is 98%, the electrical energy pulled from the utility would be:

$$= \left(\frac{9000 \text{ kWh}}{0.98} \right) = 9{,}184 \text{ kWh}$$

Then, the annual energy consumption by the gas water heater, in DT or MMBtu would be = (9184 kWh).(3412Btu/kWh)(1DT/1,000,000Btu) = **31.33 DT.**

Since the natural gas cost rate is given as $10.87/DT, the annual operating cost for the gas water heater would be = (31.33 DT).($10.87/DT) = **$340.61.**

Answer: The gas water heater would cost substantially less to operate than the electric water heater.

3. A computer manufacturing company is testing a prototype for the amount of heat it dissipates as wasted energy over a 10 hour period of operation. The computer is powered by a 24V DC power supply and is designed to draw 3A of current. Determine the total energy dissipated in Btu.

Solution:

Apply Eq. 2.10:

$$\text{Energy} = \text{V.I.t} = (24\text{V}).(3\text{A}).(10\text{hr}) = (72\text{W}).(36{,}000 \text{ s})$$

$$= \left(72 \frac{\text{J}}{\text{s}} \right)(36{,}000\text{s}) = 2{,}592{,}000 \text{ J} = 2{,}592\text{kJ}$$

Since 1.055 kJ = 1.0 BTU

$$\text{Energy Dissipated, in Btu} = (2{,}592\text{kJ})\left(\frac{1\text{Btu}}{1.055\text{kJ}}\right) = 2{,}457\text{Btu}$$

4. In response to a significant near miss incident and midair fire on a new commercial jet aircraft, a governmental agency is performing forensic analysis on the type of Lithium Ion aircraft battery suspected to be the root cause. Estimate the amount of ***current*** involved in the suspected fault on the basis of the following forensic data:

- Total energy released in the catastrophic failure of the battery: **866kJ**
- Estimated duration of fault: **2 seconds**
- Rated voltage of the battery: **$3.7V_{DC}$**

Solution:

Apply Eq. 2.10:

$$\text{Energy} = \text{V.I.t}$$

$$\text{or, I} = \frac{\text{Energy}}{\text{V.t}} = \frac{866{,}000\text{J}}{(3.7\text{V}).(2\text{s})} = \frac{866{,}000(\text{V.A.s})}{(3.7\text{V}).(2\text{s})} = 117{,}027\text{A}$$

5. A **156Sin377t** sinusoidal voltage is connected across a load consisting of a parallel combination of a **20 Ω** resistor and a 10**Ω** capacitive reactance.

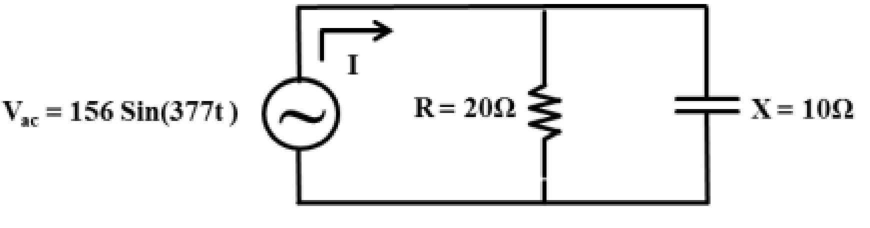
(a) Determine the real power dissipated by the resistor.

(b) Determine the reactive power stored in a **10Ω** parallel capacitive reactance.

(c) Calculate the total apparent power delivered to this parallel **R** and **X** circuit by the AC voltage source.

Solution:

The circuit diagram for this scenario would be as depicted below:



(a) We can apply Eq. 2.19 to determine the power dissipated or consumed in the 20Ω resistor. However, we must first derive the V_{RMS} from the given AC voltage of **156Sin377t.** This is due to the fact that coefficient of 156 stated in the give AC voltage function of **156Sin400t** is the peak or maximum voltage, $\mathbf{V_m}$**.**

As discussed in Segment 1 and stipulated by Eq. 1.3:

$$V_m = \sqrt{2} V_{RMS} \quad \text{or} \quad V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{RMS} = \frac{156}{\sqrt{2}} = 110.3\text{V}$$

$$P = \frac{{V_{RMS}}^2}{R} = \frac{(110.3)^2}{20} = 608 \text{ Watts}$$

(b) Apply Eq. 2.20 to determine the reactive power sequestered in the 10Ω parallel capacitive reactance.

$$Q = \frac{{V_{RMS}}^2}{X} = \frac{(110.3)^2}{10} = \frac{12{,}166}{10} = 1217 \text{ VAR}$$

(c) Apply Eq. 2.12 to calculate the total apparent power **S** delivered to this parallel R and X circuit by the AC voltage source.

$$\vec{S} = P + jQ = 608 - j1217 = 1361\angle -63.4°\Omega$$

Note that jQ reactive power entity is entered into the apparent power calculation above as "-jQ" because of the fact that capacitance in an AC circuit results in negative impedance contribution or "-jX." Therefore the reactive power Q due to a capacitor is applied as "-j1217," in overall **S** calculation.

We can also apply Eq. 2.22 to verify the magnitude of the total apparent power **S** delivered to this parallel R and X circuit:

$$|S| = \text{Magnitude of AC Apparent Power} = \sqrt{P^2 + Q^2}$$

$$= \sqrt{608^2 + 1217^2} = 1361 \text{ VA}$$

Ancillary: Reader is encouraged to verify the apparent power of 1361 VA by applying equation 4.21. **Hint:** The Z, in this case must be computed through parallel combination of R and Z_C as shown below:

$$|Z| = \left|\frac{R.Z_C}{R + Z_C}\right|$$

6. A **156Sin400t** sinusoidal voltage is connected across an unknown resistive load. If the power dissipated in the resistor is 1000 Watts, what is the resistance of the resistive load?

Solution:

We can apply Eq. 2.19 to determine the value of the resistor using the given power dissipation value of 1000W. However, we must first derive the V_{RMS} from the given AC voltage of **156Sin400t.** This is due to the fact that coefficient of 156 stated in the give AC voltage function of 156Sin400t is the peak or maximum voltage, $\mathbf{V_m}$**.**

As discussed in Segment 1 and stipulated by Eq. 1.3:

$$V_m = \sqrt{2}V_{RMS} \quad \text{or} \quad V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{RMS} = \frac{156}{\sqrt{2}} = 110.3V$$

$$P = \frac{V_{RMS}^{\ 2}}{R} \qquad \text{Eq. 4.19}$$

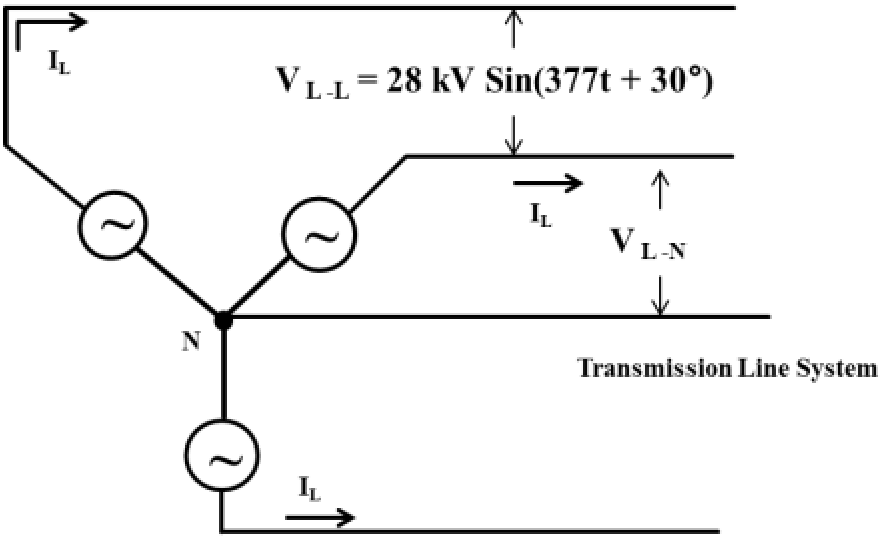
$$1000 = \frac{(110.3)^2}{R}$$

$$\therefore \quad R = 12.17\ \Omega$$

7. The AC circuit shown below depicts a three phase, one-line schematic of a hydroelectric power generating station. Assume that there is no voltage drop between the generator and the primary side of the transmission system transformer. The line current is indicated by an EMS system to be **10kA,** RMS. Calculate the following if the power factor is known to be **0.95**:

a) Magnitude of the apparent power presented to the transmission lines.

b) Magnitude of the real power presented to the transmission lines.
c) The RMS line to neutral voltage at the source.



Solution:
(a) Magnitude of the apparent power presented to the transmission lines:
Note that the AC voltage function is NOT specified in RMS form; hence, by convention, it is "peak" or "maximum" voltage. Therefore, we need to derive the RMS voltage for subsequent computations. The line current is given in RMS form.

$$\left|V_{\text{L-L},RMS}\right| = \left(\frac{V_{\text{L-L, Peak}}}{\sqrt{2}}\right) = \left(\frac{28{,}000\text{V}}{\sqrt{2}}\right) = 19{,}799\text{V}$$

According to Eq. 2.30:

$$\left|\vec{S}_{3-\phi}\right| = 3\left|\vec{S}_{1-\phi}\right| = 3\left(\frac{V_{\text{L-L}}.I_{\text{L}}}{\sqrt{3}}\right) = \sqrt{3}V_{\text{L-L}}.I_{\text{L}},\ \text{for three}$$

phase Y and Δ circuits.

Therefore,

$$\left|\vec{S}_{3-\phi}\right| = \sqrt{3}V_{\text{L-L}}.I_{\text{L}} = \sqrt{3}(19{,}799\text{V}).(10{,}000\text{A}) = 342{,}928{,}564\text{VA}$$

$$= 342{,}929\ \text{kVA} = 342.93\ \text{MVA}$$

(b) Magnitude of the real power presented to the transmission lines can be determined by rearranging and using Eq. 2.23:

$$\text{PF} = \text{Power Factor} = \frac{|P|}{|S|}$$

$$\text{or,}\quad |P| = \text{PF}.|S| = (0.95).(342.929\text{MVA}) = 326\text{MW}$$

(c) **Hint:** Use the $\mathbf{V_{L\text{-}L}}$ computed in part (a). Line to neutral voltage, as introduced in Segment 1, can be stated mathematically as:

$$V_{L\text{-}N,\,Y} = \text{Line to neutral voltage in a Y source or load configuration} = \frac{V_{L\text{-}L}}{\sqrt{3}} = \frac{19{,}799}{\sqrt{3}} = 11{,}431\text{V} = 11.43\text{kV}$$

8. A pump is to be installed on the ground floor of a commercial building to supply 200 ft^3/sec of water up to an elevation of 100 ft. Determine the minimum size of the motor for this application. Assume that the efficiency of the pump is 80%. The weight density of water $\gamma = 62.4$ lbf/ft^3

Solution

Solution strategy in this case would be to use Eq. 2.39 to compute the WHP. Then, the amount of real power "**P**" delivered by the motor would be computed based on the given efficiency of the pump.

Given:

$h_A = 100$ ft

$\dot{V} = \text{Volumetric Flow Rate} = 200\ \text{cu-ft/sec}$

Pump Efficiency = 80 %

$\gamma = 62.4$ lbf/ft^3

According to Eq. 2.39, Water Horsepower delivered by the pump to the water can be stated as:

$$\text{WHP} = P_p = \text{Fluid horse power delivered by the pump to the water}$$

$$= \frac{(h_A)(\gamma)\left(\dot{V}\right)}{550} = \frac{(100\text{ ft})\left(62.4\text{lbf/ft}^3\right)(200\text{ft}^3/\text{s})}{550\text{ ft-lbf/s/hp}}$$

$$= 2269\text{ hp}$$

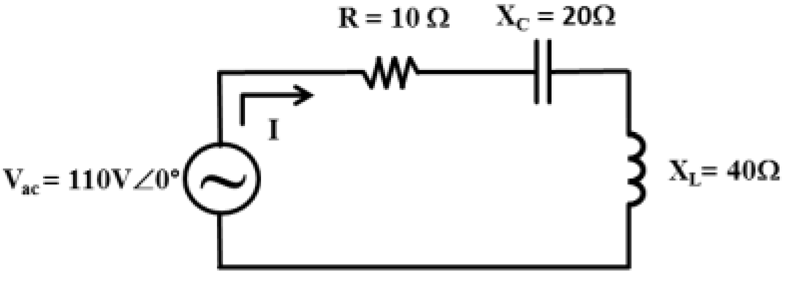
Then, according to the Wire to Water (hydraulic pump) power flow diagram in Figure 2.3:

$$\text{BHP delivered by the motor to the } = \frac{\text{WHP}}{\eta_p} = \frac{2269}{0.8}\text{ hp} = 2836\text{ hp}$$

Therefore, a commercially available motor size above 2,836 hp should be selected.

Segment 3 Solutions

1. Determine the power factor of the circuit shown below, as seen by the AC source.



Solution:
This problem can be solved through multiple approaches. Two of those approaches are as follows:

i. Calculate the equivalent or combined impedance of the circuit as seen by the source. Then take the cosine of the angle of that impedance.

ii. Calculate the equivalent or combined impedance of the circuit as seen by the source. Apply the ohm's law to compute the AC current. Then take the cosine of the angular difference between given AC voltage and the computed current.
We will utilize the first approach:

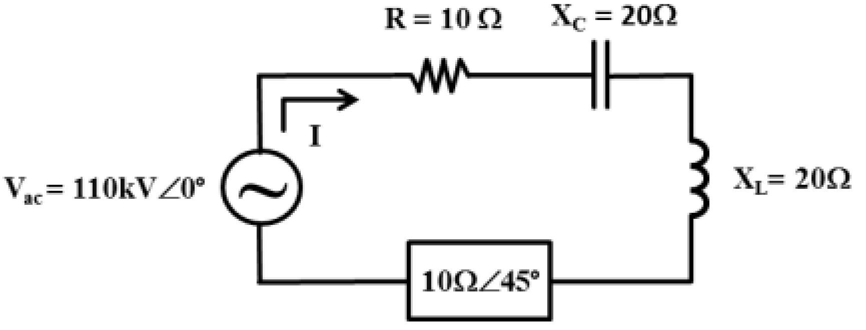
$$Z_{Eq} = 10\Omega + j20\Omega - j40\Omega = 10\Omega - j20\Omega = 22.36\angle -63.43°$$

$$\left\{ \text{Note } \left| Z_{Eq} \right| = \sqrt{10^2 + 20^2} = 22.36\ \Omega, \text{ and } \theta_Z = -\text{Tan}^{-1}\left(\frac{20}{10}\right) = -63.43 \right\}$$

Therefore, $PF = \text{Cos}\theta_Z = \text{Cos}(-63.43°) = 0.45$ or 45%

Reader is encouraged to verify the result through approach ii.

2. Assume that the circuit depicted below represents one phase of a special power transmission line. Determine the power factor of the circuit shown below, as seen by the AC source.



Solution:
As with problem 1, this problem can be solved through multiple approaches. Two of those approaches are as follows:

i. Calculate the equivalent or combined impedance of the circuit as seen by the source. Then take the cosine of the angle of that impedance.

ii. Calculate the equivalent or combined impedance of the circuit as seen by the source. Apply the ohm's law to compute the AC current. Then take the cosine of the angular difference between given AC voltage and the computed current.
In this case will illustrate the application of approach (ii):

$$Z_{Eq} = 10\Omega + j20\Omega - j20\Omega + 10\angle 45°\Omega = 10\Omega + 7.07\Omega + j7.07\Omega$$
$$= 17.07\Omega + j7.07\Omega = 18.48\Omega\angle 22.5°\Omega$$

$$\left\{ \text{Note } \left|Z_{Eq}\right| = \sqrt{17^2 + 7.07^2} = 18.48\Omega, \text{ and } \theta_Z = \text{Tan}^{-1}\left(\frac{7.07}{17.07}\right) = 22.5° \right\}$$

$$I = \frac{V}{Z_{Eq}} = \frac{110kV\angle 0°}{18.48\Omega\angle 22.5°\Omega} = 5.95kA\angle\text{-}22.5°, \text{ or } \phi_I = -22.5°$$

$$\text{Therefore, PF} = \text{Cos}(\theta_V - \phi_I) = \text{Cos}\left(0 - (-22.5°)\right) = \text{Cos}(22.5°)$$
$$= 0.923 \text{ or } 92.3\%$$

Reader is encouraged to verify the result through approach (i); which, actually, would require fewer steps.

3. If the power factor in problem 2 is less than 1.0, how much capacitance or inductance must be added in series to raise the power factor to unity?

Solution:

This problem can be solved by simply focusing on the total or equivalent impedance $\mathbf{Z_{Eq}}$ and determining the amount of reactance needed to offset the reactance in the original impedance.

$$Z_{Eq} = 10\Omega + j20\Omega - j20\Omega + 10\angle 45°\Omega = 10\Omega + 7.07\Omega + j7.07\Omega$$

$$= 17.07\Omega + j7.07\Omega = 18.48\Omega\angle 22.5°\Omega$$

Now the question is: how much and what type of reactance must be added such that Z_{Eq} reduces to its resistance portion, or 17.07Ω.

Or, $17.07\Omega + j7.07\Omega + jX\Omega = 17.07\Omega$

$\therefore \quad jX = -j7.07\Omega$, the negative sign indicates that the reactance to be added must result in negative impedance; it must be capacitive.

And, $|X| = |X_C| = 7.07\Omega$

$X_C = 7.07\Omega$, and by definition, $X_C = \dfrac{1}{\omega C}$. Frequency, f, in the US, can be assumed to be 60 hz, and by definition, $\omega = 2\pi f$. Therefore,

$$\omega = 2\pi f = 2\,(3.14)(60hz) = 377 \text{ radians/second}$$

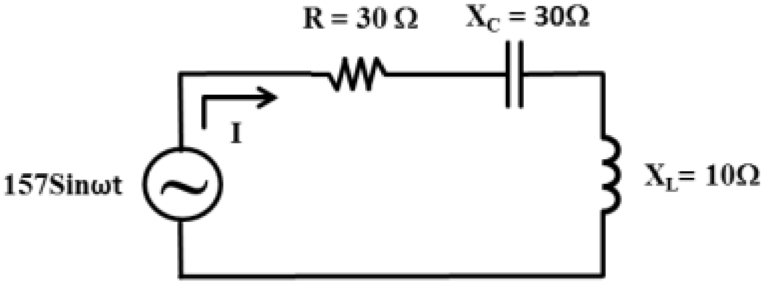
And, $C = \dfrac{1}{\omega X_C} = \dfrac{1}{(377).(7.07)} = 0.000375F$, or 375 μF

4. The output of a variable frequency drive, as shown in the circuit below, is 157Sinωt. The VFD output is currently set at 50Hz.This drive is connected to a resistive load, capacitive reactance, and an inductive reactance.

(a) What should be the new frequency setting to attain a power factor of 1, or 100%.

(b) What is the existing power factor, at 50Hz?

(c) What would be the power factor if all circuit elements remain unchanged and the VFD frequency is lowered to 30Hz.



Solution

(a) We must convert reactances $\mathbf{X_C}$ and $\mathbf{X_B}$ to corresponding capacitance, **C**, and inductance, **L**, values.

$$\text{Since } X_L = 2\pi fL,\ L = \frac{X_L}{2\pi f},\ \text{or,}\quad L = \frac{10\Omega}{2\pi(50\text{hz})} = 0.03185\ \text{H}$$

And,

$$X_C = \frac{1}{2\pi fC},\quad C = \frac{1}{2\pi fX_C},\ \text{or,}\quad C = \frac{1}{2\pi(50\text{hz})(30\Omega)} = 106\text{x}10^{-6}\text{F}$$

$$f = f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2(3.14)\sqrt{(0.03185).(106\text{x}10^{-6})}} \qquad \textbf{Eq. } 3.8$$

$$\therefore\quad f = f_o = 86\text{hz}$$

(b) The power factor can be calculated through the impedance angle, using equation 5.3. We must compute the circuit's total impedance first***, at 50 Hz***.

As derived in part (a), L = 0.03185H and C = 0.000106F

$$\therefore\quad X_L = 2\pi fL = 2(3.14)(50\text{hz})(0.03185\text{H}) = 10\Omega \quad \{\text{Given}\}$$

$$\text{And, } X_C = \frac{1}{2\pi fC} = \frac{1}{2(3.14)(50\text{hz})(0.000106\text{F})} = 30\Omega \quad \{\text{Given}\}$$

$$X_t = X_Z =$$

$$\therefore\ Z = 30\Omega - j30\Omega + j10\Omega = 30\Omega - j20\Omega = 36\angle{-33.7}°\Omega$$

And, Power Factor = $\text{Cos}\,(\theta_Z) = \text{Cos}\,(-33.7°) = 0.83.3$,

or 83.3% leading.

(c) The power factor at 30Hz:

As derived in part (a), L = 0.03185H and C = 0.000106F

$\therefore \quad X_L = 2\pi fL = 2(3.14)(30hz)(0.03185H) = 6\Omega$

$$\text{And, } X_C = \frac{1}{2\pi fC} = \frac{1}{2(3.14)(30hz)(0.000106F)} = 50\Omega$$

$\therefore \; Z = 30\Omega - j50\Omega + j6\Omega = 30\Omega - j44\Omega = 53.3\angle{-55.7°}\Omega$

And, Power Factor = $\text{Cos}\,(\theta_Z) = \text{Cos}\,(-55.7°) = 0.5631$,

or 56.31% leading.

5. The HMI (Human Machine Interface) monitor of an Automated HVAC system, monitoring an air washer supply fan motor is indicating a reactive power, $\mathbf{Q_1}$, of 60kVARs. This system is located in the United Kingdom, where the AC frequency is 50 Hz. Determine the amount of capacitance that must be added to improve the power factor of the motor branch circuit such that the reactive power is reduced to $\mathbf{Q_2}$ of 20kVARs. The branch circuit is operating at 240 V_{RMS}.

Solution:

Apply Eq. 3.4:

$$C = \frac{(Q_1 - Q_2)}{2\pi fV^2} = \frac{(60kVAR - 20kVAR)}{2\pi(50hz)(240V)^2}$$

$$= \frac{(60{,}000VAR - 20{,}000VAR)}{2\pi(50hz)(240V)^2} = 0.002211F = 2.211mF$$

6. Power Factor Improvement and Cost Savings: In conjunction with the local utility company DSM program, a manufacturing plant is being offered \$2 per kVA for improvement in power factor from 0.75 to 0.85. The plant is operating at its contract level of 30 MW. Determine estimated annual pre-tax revenue if the plant accepts the offer.

Solution:

Since the objective is to assess the cost savings on the basis of apparent power (**S**) reduction, we must determine the apparent power $\mathbf{S_1}$ being drawn by the air compressor motor at the existing power factor of 0.75 (75%), and the apparent power $\mathbf{S_2}$ at the desired power factor of 0.85 (85%). Rearrange and apply Eq. 3.1.

$$\text{Magnitude of Apparent Power } |S| = \frac{|P|}{PF}$$

$$\text{Magnitude of existing apparent power } |S_1| = \frac{|P|}{PF_1} = \frac{30{,}000\text{kW}}{0.75}$$

$$= 40{,}000\text{kVA}$$

$$\text{Magnitude of improved apparent power } |S_2| = \frac{|P|}{PF_2} = \frac{30{,}000\text{kW}}{0.85}$$

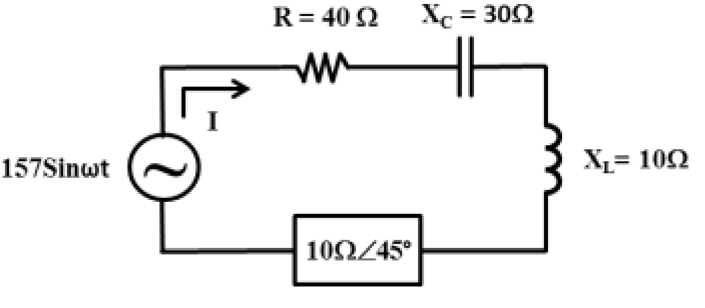
$$= 35{,}294\text{kVA}$$

Annual DSM revenue expected from proposed power factor improvement

$$= (40{,}000\text{kVA} - 35{,}294\text{kVA}).\left(\frac{\$2}{\text{kVA - Month}}\right).(12\text{ Months/Year})$$

$$= \$112{,}941$$

7. The output of a variable frequency drive, as shown in the circuit below, is 157Sinωt. The VFD output is currently set at 60Hz.This drive is connected to a resistive load, capacitive reactance, inductive reactance and a "black box" load, $\mathbf{Z_B}$, of 10Ω∠45°. What should be the new frequency setting to attain a power factor of 1, or 100%?



Solution

Solution strategy**:** As explained earlier in this section, the power factor of an AC circuit, consisting of inductive and capacitive reactance, peaks to the maximum value of unity, or 100%, at resonance frequency, $\mathbf{f_0}$. However, in this case, because of the presence of the black box impedance of $\mathbf{Z_B}$ =10Ω∠45°, we cannot apply Eq. 3.5, directly, to compute $\mathbf{f_0}$. We must convert impedance into its rectangular form to derive the reactance component $\mathbf{X_{L,B}}$, combine it with the other inductive reactance in the circuit, derive the L and C values and then apply Eq. 3.5 to calculate the resonance frequency $\mathbf{f_0}$.

$Z_B = 10\angle 45° = 7.07\Omega + j7.07\Omega$. Therefore, $X_{L,B} = 7.07\Omega$.

Since the other inductive reactance is contributed by $X_L = 10\Omega$,

$$X_{L,total} = X_L + X_{L,B} = 10\Omega + 7.07\Omega = 17.07\Omega$$

$\therefore$ Total "equivalent L" in the circuit wuld be:

$$X_{L,total} = 2\pi f L_{total} = 17.07\Omega,$$

$$\therefore L_{total} = \frac{17.07\Omega}{2\pi f} = \frac{17.07\Omega}{2(3.14)(60hz)}, \text{ or,} \quad L = 0.0453H$$

$$\text{And, } X_C = \frac{1}{2\pi f C} = \frac{1}{2(3.14)(60hz)(C)} = 30\Omega$$

$$\therefore C = \frac{1}{2(3.14)(60hz)(30\Omega)} = 88.46 \text{ x } 10^{-6}F$$

Then, according to Eq. 3.8,

$$f_0 = \frac{1}{2\pi\sqrt{L.C}} = \frac{1}{2(3.14)\sqrt{(0.0453)(88.46 \text{ x } 10^{-6})}} = 79.54hz$$

Appendix B

Common Units and Unit Conversion Factors

Power

In the SI or Metric unit system, DC power or “real” **power** is traditionally measured in watts and:

kW = 1,000 Watts
MW = 1,000,000 Watts = 10^{6} W
GW = 1,000,000,000 Watts = 10^{9} W
TW = 10^{12} W

Where k = 1000, M = 1000,000, G = 1 billion, and T = 1 trillion.

Some of the more common **power conversion factors** that are used to convert between SI System and US system of units are listed below:

1.055 kJ/s = 1.055 kW = 1 BTU/s
1–hp = One hp = 746 Watts
= 746 J/s
= 746 N-m/s
= 0.746 kW
= 550 ft-lbf/sec

Energy

In the SI or Metric unit system, DC energy or “**real” energy** is traditionally measured in Wh, kWh, MWh, GWh, TWh (10^{12} Wh).

kWh = 1,000 Watt-hours
MWh = 1,000,000 Watt-hour = 10^{6} Wh
GWh = 1,000,000,000 Watt-hours = 10^{9} Wh
TWh = 10^{12} Wh

Some mainstream conversion factors that can be used to convert electrical energy units within the SI realm or between the SI and US realms are referenced below:

1000 kW x 1h = 1 MWh
1 BTU = 1055 J = 1.055 kJ
1 BTU = 778 ft-lbf

Energy, Work and Heat Conversion Factors:

Energy, Work or Heat		
Btu	1.05435	kJ
Btu	0.251996	kcal
Calories (cal)	4.184	Joules (J)
ft-lbf	1.355818	J
ft-lbf	0.138255	kgf-m
hp-hr	2.6845	MJ
KWH	3.6	MJ
m-kgf	9.80665	J
N-m	1	J

Power Conversion Factors:

Power		
Btu/hr	0.292875	Watt (W)
ft-lbf/s	1.355818	W
Horsepower (hp)	745.6999	W
Horsepower	550.*	ft-lbf/s

Temperature Conversion Factors/Formulas:

Temperature		
Fahrenheit	(°F-32) /1.8	Celsius
Fahrenheit	°F+459.67	Rankine
Celsius	°C+273.16	Kelvin
Rankine	R/1.8	Kelvin

Common Electrical Units, their components and nomenclature:

Force	Newton	**N**	**kg m s^{-2}**
Energy	joule	**J**	**kg m^2 s^{-2}**
Power	watt	**W**	**kg m^2 s^{-3}**
Frequency	hertz	**Hz**	**s^{-1}**
Charge	coulomb	**C**	**A s**
Capacitance	farad	**F**	**C^2 s^2 kg^{-1} m^{-2}**
Magnetic Induction	tesla	**T**	**kg A^{-1} s^{-2}**

Common Unit Prefixes:

1.00E-12	pico	p
1.00E-09	nano	n
1.00E-06	micro	µ
1.00E-03	milli	m
1.00E+03	kilo	k
1.00E+06	mega	M
1.00E+09	giga	G
1.00E+12	tera	T

Wire Size Conversions:

A circular mil can be defined as a unit of area, equal to the area of a circle with a diameter of one mil (one thousandth of an inch), depicted as:

1 circular mil is approximately equal to:

- 0.7854 square mils (1 square mil is about 1.273 circular mils)
- 7.854×10^{-7} square inches (1 square inch is about 1.273 million circular mils)
- 5.067×10^{-10} m^2
- 506.7 μm^2

1000 circular mils = 1 MCM or 1 kcmil, and is (approximately) equal to:

- 0.5067 mm^2, so 2 kcmil ≈ 1 mm^2

AWG to Circular Mil Conversion

The formula to calculate the circular mil for any given AWG (American Wire Gage) size is as follows:

.$\boldsymbol{A_n}$ represents the circular mil area for the AWG size $\boldsymbol{n}$.

$$A_n = (5 \times 92^{\frac{36-n}{39}})^2$$

For example, a AWG number 12 gauge wire would use **n = 12**; and the calculated result would be 6529.946789 circular mils

Circular Mil to mm^2 and Dia (mm or in) Conversion:

kcmil or, MCM	mm²	Diameter in.	Diameter mm
250	126.7	0.5	12.7
300	152	0.548	13.91
350	177.3	0.592	15.03
400	202.7	0.632	16.06
500	253.4	0.707	17.96
600	304	0.775	19.67
700	354.7	0.837	21.25
750	380	0.866	22
800	405.4	0.894	22.72
900	456	0.949	24.1
1000	506.7	1	25.4
1250	633.4	1.118	28.4
1500	760.1	1.225	31.11
1750	886.7	1.323	33.6
2000	1013.4	1.414	35.92

Appendix C – Greek Symbols Commonly Used in Electrical Engineering

Greek Alphabet			
Αα	Alpha	Νν	Nu
Ββ	Beta	Ξξ	Xi
Γγ	Gamma	Οο	Omicron
Δδ	Delta	Ππ	Pi
Εε	Epsilon	Ρρ	Rho
Ζζ	Zeta	Σσς	Sigma
Ηη	Eta	Ττ	Tau
Θθ	Theta	Υυ	Upsilon
Ιι	Iota	Φφ	Phi
Κκ	Kappa	Χχ	Chi
Λλ	Lambda	Ψψ	Psi
Μμ	Mu	Ωω	Omega